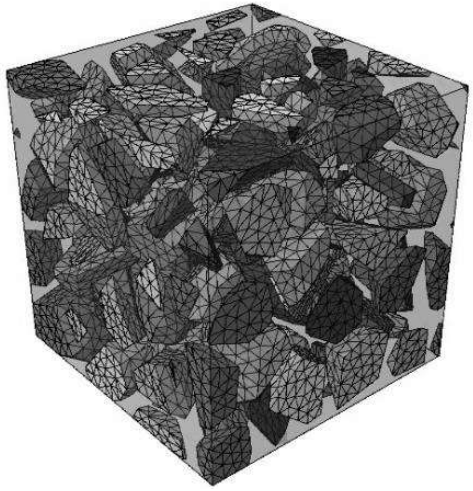
5. Anizotropik Malzemelerin Elastik Davranışları

Elastic Behaviours of Anisotropic Materials (Anistropic Elasticity)



Kompozit malzemeler aynı zamanda anizotropik malzemelerdir ve bunların daha özelleştirilmiş halleridir.

Kompozitlerin mekaniği anlayabilmek için öncelikle daha genel olan anizotropik malzemelerin mekaniğini bilmek önemlidir.

Bu bölümde Amacımız:

En genelden başlayarak özele doğru anizotropik malzemelerin tiplerini anlatmak; herbir tip için elastik durumda gerilme-şekil değiştirme arasındaki ilişkileri sağlayan malzeme sabitlerini ifade etmektir.

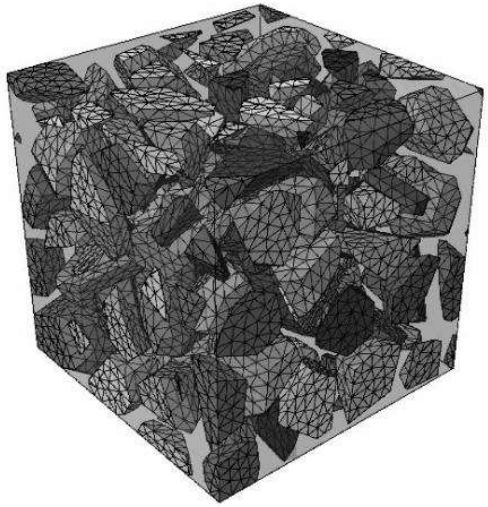
5.1 Anizotropik Malzeme Nedir?

What is the Anisotropic Material?

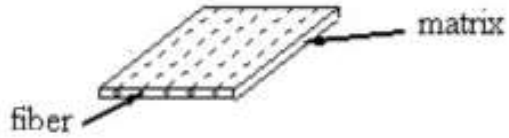
Anizotropik Malzeme:
Anisotropic Materials:

Homojen olmayan ve Isıl veya mekanik yüklemelere karşı, doğrultuya ve yöne göre farklı mekanik davranış gösteren malzemelerdir. (are nonhomogeneous materials and have the different mechanical behaviours according to the directions, for thermal and mechanical loadings.

- İç yapı tane dağılımı olarak homojen değildir.
  (Their internal structure are not homogeneous.)
- Fiber takviyeli Kompozit Malzemeler , Aniztropik malzemelerin özel bir halidir.
  (Fiber reinforced composites are the special case of the anistropic materials.)
- Fiber doğrultusu ve fiberlere dik doğrultulara göre malzeme sabitlerinin belirlenmesi yeterlidir. Bu tip malzemelere ortotropik malzemeler denir.
  (Detarmination of the properties along and perpendicular to the fiber direction are enough. This type of materials are called orthtoropic)
- Transverly anizotropik ve monoklinik malzemeler de aniztropik malzemelerin özel halleridir. (bunlar ileride açıklanacaktır)
- Aslında izotropik malzemeler, anizotropik malzemenin en özel halidir. (In fact, isotropic materials are the most special case of the anistiropic materials)



En genel anizotropik malzeme modeli

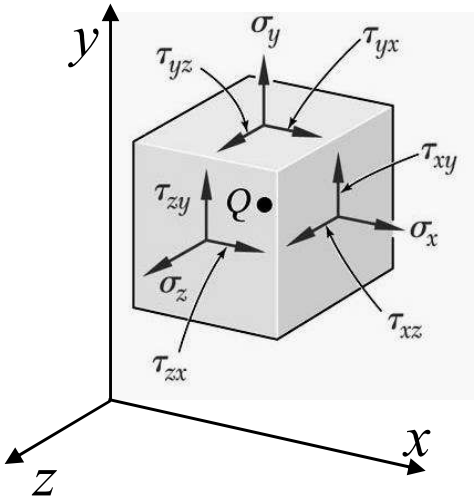


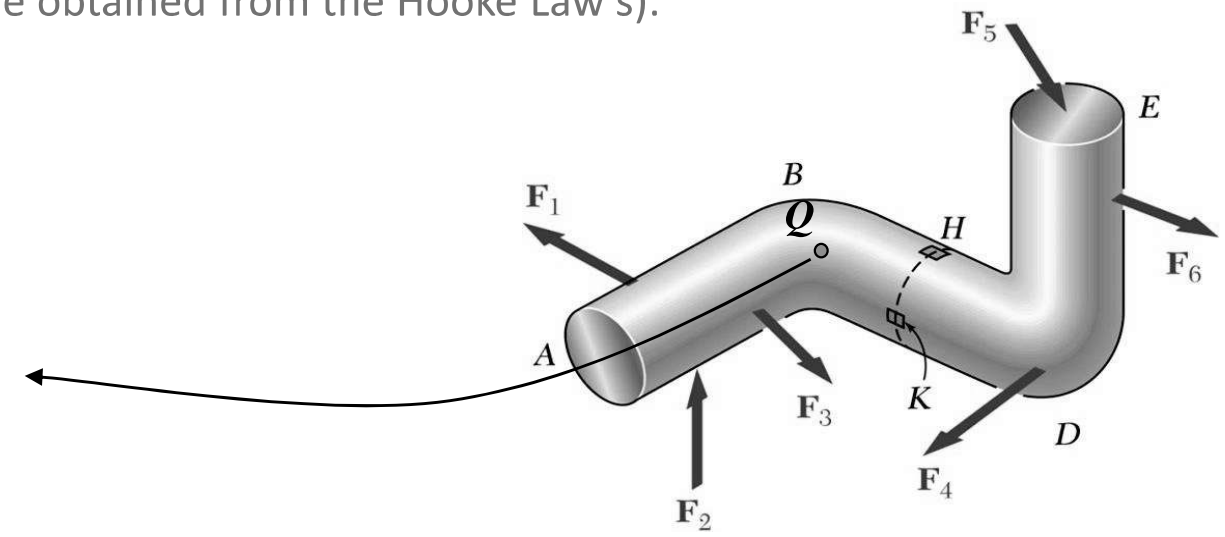
Kompozit bir tabaka

5.2 İzotropik malzemeler için En genel durumda Hooke bağıntılarını hatırlayalım:

Bir noktada en genel durumda 6 tane bağımsız gerilme bileşeni vardır (σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{xz}).
(In general loaidng situation, there are independent six stress components at a point)

İzotropik malzemeler için bu noktadaki elastik şekil değiştirmeler alltaki Hooke bağıntılarından elde edilir.
(Elastic strains for isotropic materials are obtained from the Hooke Law's):





Kaymadaki Şekil değiştirmeler

Sıcaklık etkisi:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha.\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha.\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] + \alpha.\Delta T$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G}$$

Matris formatında Hooke Bağıntılarını yazacak olursak;

Matris forms of Hooke equations:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\upsilon)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\upsilon)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\upsilon)}{E} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_x \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix}$$

Veya şekil değiştirmeler cinsinden gerilmeleri yazalım.

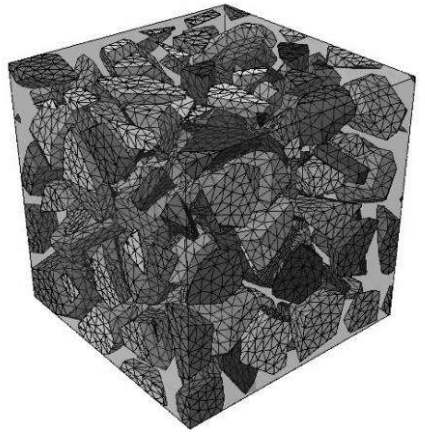
$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0\\ \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0\\ \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{E}{2(1+\upsilon)}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}$$

Görüldüğü gibi izotropik malzemelerde sadece 2 tane elastik malzeme sabiti (E, ν) vardır.

As you can see, there are two material constants for isotropic materias (E, ν)

ANİZTROPİK malzemelerde Durum Nasıl olur?

How is the case at Anistropic Materials?



ACABA

Malzememiz izotropik değilse, yani aniztropik ise

Gerilme – gerinme bağıntıları (Hooke Bağıntıları) nasıl olur? Malzeme Sabitleri Ne olur?

If our material ıs not ısotropic (if ıt ıs anıstrropic);

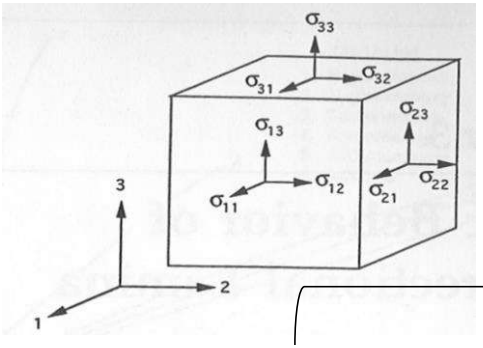
How do the stress-straın relatıons change? What are the Material Constants?

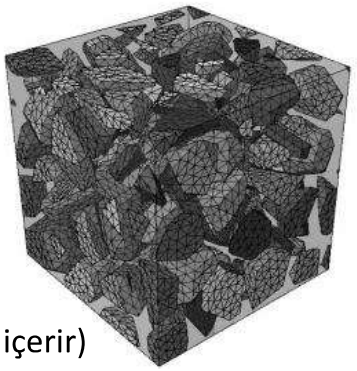
Bundan sonra bu soruların cevapları aranacaktır

From now on, the answers of these questıons will be searched

5.3 En Genel Anizotropik Malzeme için Elastik Malzeme Sabitleri

Elastic Material Constants of Most General Anistropic Material





Rijitlik (Stiffness) matrisi: [C]

(9x9= 81 bağımsız malzeme sabiti içerir)

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \\ \sigma_{32} \\ \sigma_{13} \\ \sigma_{21} \end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1123} & C_{1131} & C_{1112} & C_{1132} & C_{1113} & C_{1121} \\ C_{2211} & C_{2222} & C_{2233} & C_{2223} & C_{2231} & C_{2212} & C_{2232} & C_{2213} & C_{2221} \\ C_{3311} & C_{3322} & C_{3333} & C_{3323} & C_{3331} & C_{3312} & C_{3332} & C_{3313} & C_{3321} \\ C_{2311} & C_{2322} & C_{2333} & C_{2323} & C_{2331} & C_{2312} & C_{2332} & C_{2313} & C_{2321} \\ C_{3111} & C_{3122} & C_{3133} & C_{3123} & C_{3131} & C_{3112} & C_{3132} & C_{3113} & C_{3121} \\ C_{1211} & C_{1222} & C_{1233} & C_{1223} & C_{1231} & C_{1212} & C_{1232} & C_{1213} & C_{1221} \\ C_{3211} & C_{3222} & C_{3233} & C_{3223} & C_{3231} & C_{3212} & C_{3232} & C_{3213} & C_{3221} \\ C_{1311} & C_{1322} & C_{1333} & C_{1323} & C_{1331} & C_{1312} & C_{1332} & C_{1313} & C_{1321} \\ C_{2111} & C_{2122} & C_{2133} & C_{2123} & C_{2131} & C_{2112} & C_{2132} & C_{2113} & C_{2121} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{23} \\ \epsilon_{31} \\ \epsilon_{12} \\ \epsilon_{32} \\ \epsilon_{13} \\ \epsilon_{21} \end{bmatrix}$$

İndis formatında

$$\sigma_{ij} = C_{ijkl}\,\epsilon_{kl}$$

$(i, j, k, l = 1, 2, 3)$

C_{ijkl} = Stiffness components

Gevşeklik (Compliance) Matrisi: [S] *Rijitlik matrisinin tersidir.* $[S]=[C]^{-1}$

$$\begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{23} \\ \epsilon_{31} \\ \epsilon_{12} \\ \epsilon_{32} \\ \epsilon_{13} \\ \epsilon_{21} \end{bmatrix} = \begin{bmatrix} S_{1111} & S_{1122} & S_{1133} & S_{1123} & S_{1131} & S_{1112} & S_{1132} & S_{1113} & S_{1121} \\ S_{2211} & S_{2222} & S_{2233} & S_{2223} & S_{2231} & S_{2212} & S_{2232} & S_{2213} & S_{2221} \\ S_{3311} & S_{3322} & S_{3333} & S_{3323} & S_{3331} & S_{3312} & S_{3332} & S_{3313} & S_{3321} \\ S_{2311} & S_{2322} & S_{2333} & S_{2323} & S_{2331} & S_{2312} & S_{2332} & S_{2313} & S_{2321} \\ S_{3111} & S_{3122} & S_{3133} & S_{3123} & S_{3131} & S_{3112} & S_{3132} & S_{3113} & S_{3121} \\ S_{1211} & S_{1222} & S_{1233} & S_{1223} & S_{1231} & S_{1212} & S_{1232} & S_{1213} & S_{1221} \\ S_{3211} & S_{3222} & S_{3233} & S_{3223} & S_{3231} & S_{3212} & S_{3232} & S_{3213} & S_{3221} \\ S_{1311} & S_{1322} & S_{1333} & S_{1323} & S_{1331} & S_{1312} & S_{1332} & S_{1313} & S_{1321} \\ S_{2111} & S_{2122} & S_{2133} & S_{2123} & S_{2131} & S_{2112} & S_{2132} & S_{2113} & S_{2121} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \\ \sigma_{32} \\ \sigma_{13} \\ \sigma_{21} \end{bmatrix}$$

İndis formatında

$$\epsilon_{ij} = S_{ijkl}\,\sigma_{kl}$$

$(i, j, k, l = 1, 2, 3)$

S_{iikl} = Compliance components

Gerilme, gerinme, rijitlik ve gevşeklik tansörleri için, aşağıdaki gibi, kısaltılmış notasyonlar kullanmak kompozit malzemeler için daha kolaydır ve bu mekanikte yaygınlaşmıştır.

We can also use these notations instead of the matrixes.

$$\sigma_{11} = \sigma_1$$

$$\sigma_{22} = \sigma_2$$

$$\sigma_{33} = \sigma_3$$

$$\sigma_{23} = \tau_{23} = \sigma_4 = \tau_4$$

$$\sigma_{31} = \tau_{31} = \sigma_5 = \tau_5$$

$$\sigma_{12} = \tau_{12} = \sigma_6 = \tau_6$$

$$\epsilon_{11} = \epsilon_1$$

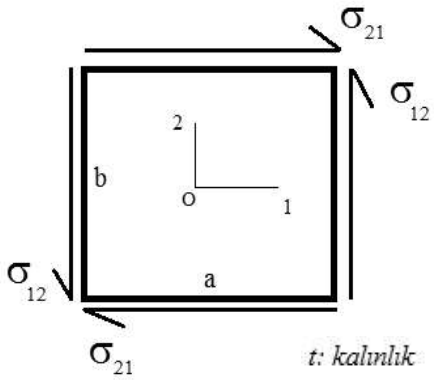
$$\epsilon_{22} = \epsilon_2$$

$$\epsilon_{33} = \epsilon_3$$

$$2\epsilon_{23} = \gamma_{23} = \epsilon_4 = \gamma_4$$

$$2\epsilon_{31} = \gamma_{31} = \epsilon_5 = \gamma_5$$

$$2\epsilon_{12} = \gamma_{12} = \epsilon_6 = \gamma_6$$



$$\Sigma M_o = 0$$

$$2.\sigma_{12}.b.t.\frac{a}{2} - 2.\sigma_{21}.a.t.\frac{b}{2} = 0$$

$$\Rightarrow \quad \sigma_{12} = \sigma_{21}$$

1-3 ve 2-3 düzlemleri içinde aynı analiz yapılabilir

$$\sigma_{23} = \sigma_{32},$$

$$\sigma_{13} = \sigma_{23}$$

Sonuçta, genel olarak; $\sigma_{ij} = \sigma_{ji}$ yazılabilir.
Generally, we can write as

Aynı düzlemde kayma gerilmelerinden kaynaklanan şekil değiştirmeler için indislerin yeri farketmez.

We can also write

$$\varepsilon_{ij} = \varepsilon_{ji}$$

Bu durumda aniztropik bir malzemede rijitlik veya gevşeklik matrsilerindeki bağımsız elastik malzeme sabitleri 6x6=36 ya indirgenmiş olur. (In this case, the material constant at the stiffness or complience matrix are reduced as 6x6=36)

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{bmatrix}$$

*Böylece, anizotropik bir cisim için kısaltılmış notasyonlar ile **gerilme-gerinme** ve **gerinme-gerilme** bağıntıları yandaki gibi gösterilebilir:*

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix}$$

Veya indis notasyonuyla :

$$\sigma_i = C_{ij}\, \epsilon_j$$

$$\epsilon_i = S_{ij}\, \sigma_j$$

$$(i, j = 1, 2, 3, \ldots, 6)$$

5.4 Triklinik Malzeme (Triclinics Materials)



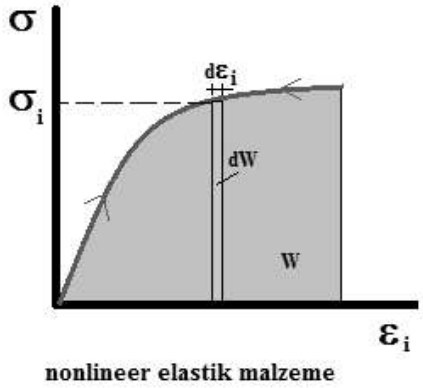
Nonlineer elastic – anizotropik bir malzemede depolanan enerji yoğunluğu:dW ise,

The energy density at a nonlineer elastic-anisotropic material :dW

$$dW = \sigma_i d\varepsilon_i$$

$$\sigma_i = C_{ij}\varepsilon_j$$

$$dW = C_{ij}\varepsilon_j d\varepsilon_i$$



nonlineer elastik malzeme

$$\frac{\partial W}{\partial \varepsilon_i} = C_{ij}\varepsilon_j \rightarrow \frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j} = C_{ij} \qquad (I)$$

Gerilme-gerinme bağıntısı

$\sigma_j = C_{ji}\varepsilon_i$ Şeklinde de yazılabilir.

Bu durumda benzer şekilde:

$$\partial W = \sigma_j \partial \varepsilon_j \rightarrow \partial W = C_{ji}\varepsilon_i \partial \varepsilon_j$$

$$\frac{\partial W}{\partial \varepsilon_j} = C_{ji}\varepsilon_i \rightarrow \frac{\partial^2 W}{\partial \varepsilon_j \partial \varepsilon_i} = C_{ji} \qquad (II)$$

(I) ve (II) denklemleri birbirine eşit olduğundan

$$\frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j} = \frac{\partial^2 W}{\partial \varepsilon_j \partial \varepsilon_i}$$

$$C_{ij} = C_{ji}$$

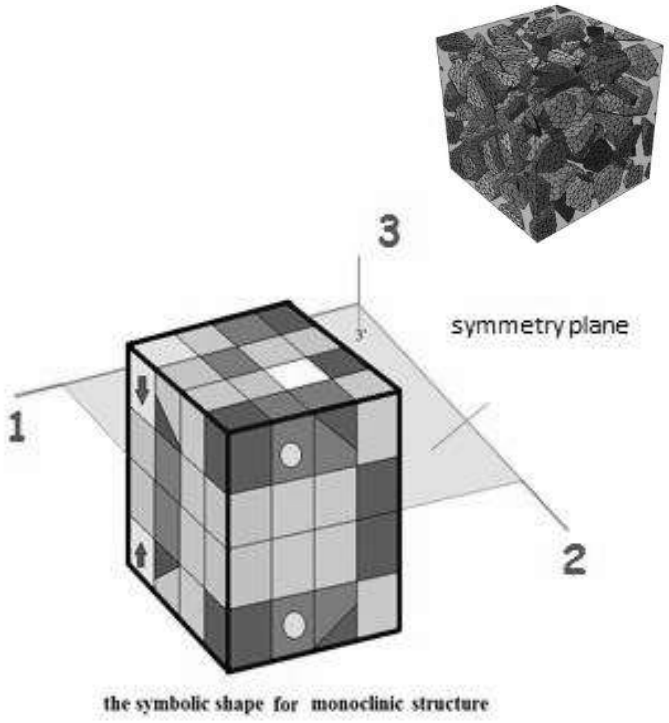
$$S_{ij} = S_{ji}$$

Bu durumda $\boldsymbol{C}$ ve $\boldsymbol{S}$ rijitlik ve gevşeklik matrisleri diyagonale göre simetrik olmalıdır. Bu matrislerdeki bağımsız, elastik malzeme sabitleri 21 dir. **Malzeme özellikleri bakımından simetri düzlemleri olmayan böyle bir anizotropik malzemeye Triklinik malzeme denir.** (In this case, the number of independent material constants is 21. This type of material is called triclinic.)

5.5 Monoklinik Malzeme: (Monoclinics Materials)

If there is an one symmetry plane of materials :

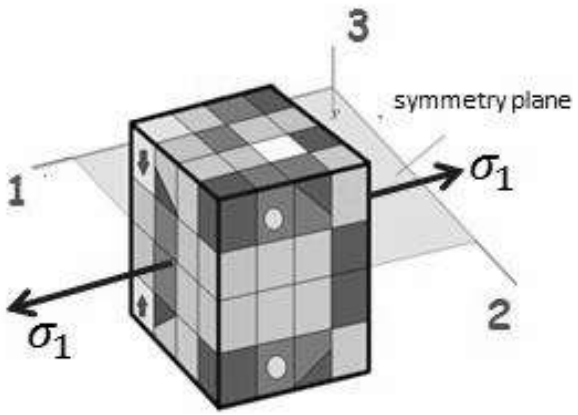
- Anizotropik bazı malzemelerin iç yapısında, yapıyı teşkil eden kristal veya daha büyük makro hacimler seviyesinde bir simetri düzlemi vardır ve fiziksel malzeme özelikleri o düzleme göre o noktada simetriktir. (There is a plane of symmetry on each point at some materials having crystal structure. (Examp: Potasium Feldspar))
- Homojen yapılarda tüm noktalardaki bu simetri düzlemleri birbirlerine paraleldir. (These symmetry planes are parallel to each other at the homegenus structures)
- Bu tip malzemelere **monoklinik malzemeler** denir. (This type of materials is called monoclic)
- Bunlara örnek olarak seramik maddelerin hammadesi olan «feldspar» gösterilebilir.
- Genel anlamda Simetri düzleminin tanımlanabildiği malzemenin en küçük parçası için monoklinik özellikten bahsedilir.



the symbolic shape for monoclinic structure

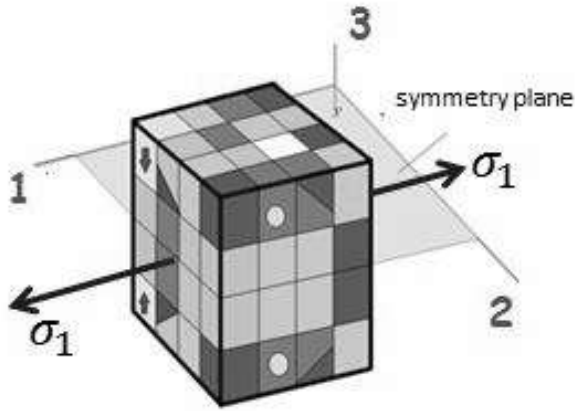
Şekilde, 1-2 (x-y) düzlemine göre simetrik olan monoklinik malzeme sembolize edilmiştir. (Herbir küçük kübün bu düzleme göre simetrik olduğuna dikkat ediniz.)


Potasyum feldspar



- Tabakalı Kompozit malzemeler, kristal seviyesinden çok daha büyük ölçülerde, (gözle görülebilecek makro seviyede) simetri düzlemline veya birbirlerine paralel simetri düzlemlerine sahip olabilirler. (The laminated composites have the symmmetry planes at macro level.)
- Bu sebeple bir sonraki sayfada anlatılacak monoklinik malzemeler için yapılan indirgemeler bu tip kompozitler için ortalama olarak yapılabilir. (Therefore the reduction for monclinic materials, explained at the next pages, can approximatly be done for the composites.)
- Burada önemli olan, malzeme özelliklerini belirlemek için yapılacak testlerde, test numunelerinin de simetri düzlemini içerecek büyüklükte olmasıdır. (The most important point for the composites is that the test speciements also include the plane of symmetry)

5.5.1 Monoklinik malzemelerde Elastik Malzeme Sabitleri



1 düzleme göre malzeme simetrisi:

Anizotropik, elastik ve monoklinik bir yapının, bir noktasında 1-2 düzlemine göre malzeme simetrisi olduğunu düşünüyoruz (Let's consider there is a plan of material symmetry for the plane 1-2 at any point of a structure that is anistropic, elastic and monoclinic.) :

Bu noktada sadece 1 doğrultusunda normal gerilme (σ_1) uygulanırsa, simetriden dolayı 1-3 ve 2-3 doğrultularında çarpılma oluşmayacaktır. Yani:

$\gamma_4 = \gamma_5$=0 olur.

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_4(=0) \\ \gamma_5(=0) \\ \gamma_6 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\gamma_4 = s_{41}\sigma_1 = 0$
$\rightarrow s_{41}$=0 $\rightarrow s_{14}$ =0

$\gamma_5 = s_{51}\sigma_1 = 0$
$\rightarrow s_{51}$=0$\rightarrow s_{15}$=0

Benzer şekilde sadece σ_2 uygulanırsa yine 1-3 ve 2-3 doğrultularında çarpılma olmaz.

$\gamma_4 = s_{42}\sigma_2 = 0 \rightarrow s_{42}$=0$\rightarrow s_{24}$=0

$\gamma_5 = s_{52}\sigma_2 = 0 \rightarrow s_{52}$=0$\rightarrow s_{25}$=0

Sadece σ_3 uygulanırsa

$\gamma_4 = s_{43}\sigma_3 = 0 \rightarrow s_{43}$=0$\rightarrow s_{34}$=0

$\gamma_5 = s_{53}\sigma_3 = 0 \rightarrow s_{53}$=0$\rightarrow s_{35}$=0

bulunur.

Bu durumda, monoklinik ve elastik bir malzeme için gevşeklik ve rijitlik matrislerinde bağımsız malzeme sabitleri 13'e indirgenmiş olur.:

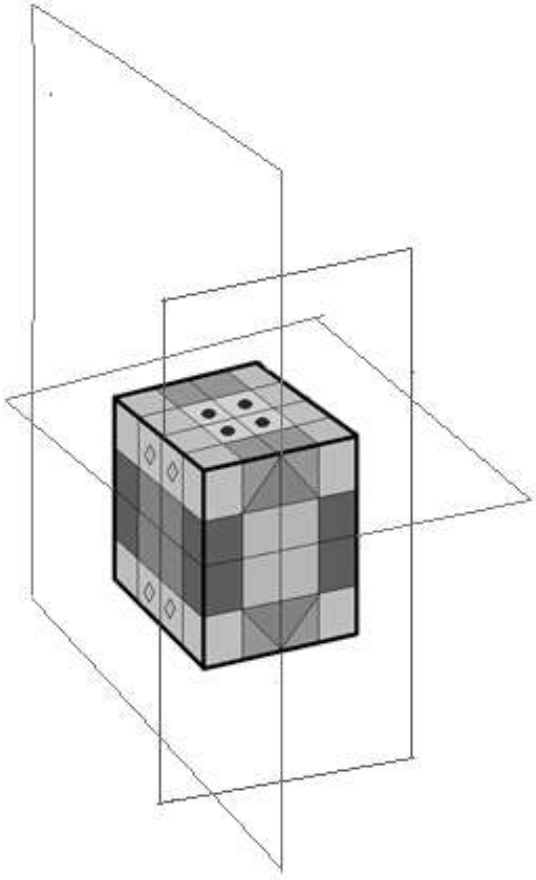
$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{21} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{31} & S_{32} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{54} & S_{55} & 0 \\ S_{61} & S_{62} & S_{63} & 0 & 0 & S_{66} \end{bmatrix}$$

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}$$

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{21} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{31} & S_{32} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{54} & S_{55} & 0 \\ S_{61} & S_{62} & S_{63} & 0 & 0 & S_{66} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\upsilon_{21}}{E_2} & -\frac{\upsilon_{31}}{E_3} & 0 & 0 & \frac{\upsilon_{61}}{G_{12}} \\ -\frac{\upsilon_{12}}{E_1} & \frac{1}{E_2} & -\frac{\upsilon_{32}}{E_3} & 0 & 0 & \frac{\upsilon_{62}}{G_{12}} \\ -\frac{\upsilon_{13}}{E_1} & -\frac{\upsilon_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & \frac{\upsilon_{63}}{G_{12}} \\ 0 & 0 & 0 & \frac{1}{G_{23}} & \frac{\upsilon_{54}}{G_{13}} & 0 \\ 0 & 0 & 0 & \frac{\upsilon_{45}}{G_{23}} & \frac{1}{G_{13}} & 0 \\ \frac{\upsilon_{16}}{E_1} & \frac{\upsilon_{26}}{E_2} & \frac{\upsilon_{36}}{E_3} & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

5.6 Ortotropik Malzeme ve Elastik Malzeme Sabitleri

(orthotropic material and elastic material constants)



Eğer Malzemenin herbir noktasında, 1-2, 2-3 ve 1-3 düzlemlerinin her üçüne göre simetri varsa, bu tip malzemelere ortotropik malzemeler denir. (If there are 3 planes of symmetry, for plane 1-2, 23-, 1-3, these materials are called orthtropic material.)

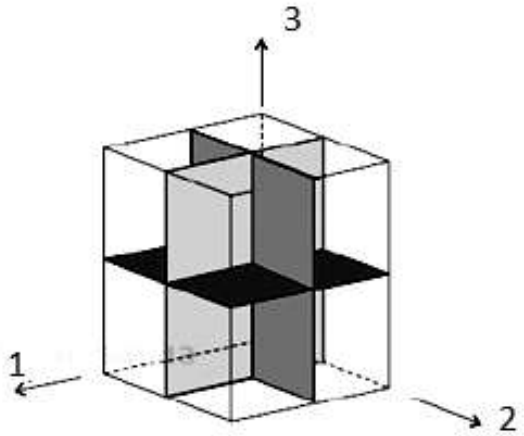


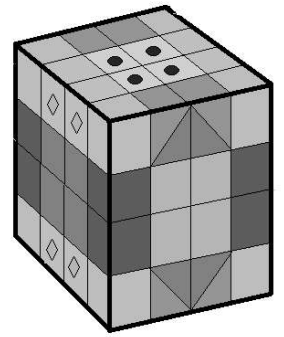
Figure 2.11: Material with three planes of symmetry.

5.6 Ortotropik Malzeme ve Elastik Malzeme Sabitleri -devam

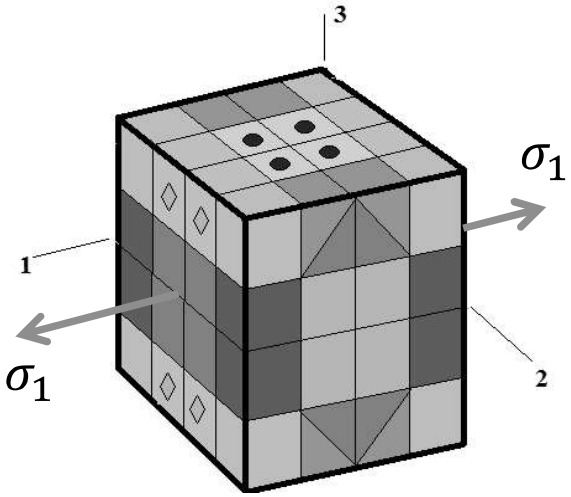
(orthotropic material and elastic material constants -continue)

Aynı şekilde sadece 1 (x) yönünde normal gerilme (σ_1) olursa, simetri düzlemlerinin hiçbirinde çarpılma meydana gelmeyecektir.

$$\gamma_4 = \gamma_5 = \gamma_6 = 0$$



γ_4 ve γ_5= 0 monoklinik malzeme özelliği idi. Bunlara ilaveten gelen γ_6 =0 denklemi için;



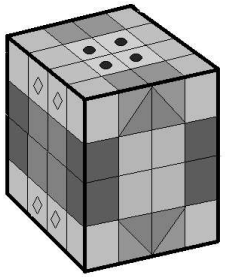
$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_4(=0) \\ \gamma_5(=0) \\ \gamma_6(=0) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{21} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{31} & S_{32} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{54} & S_{55} & 0 \\ S_{61} & S_{62} & S_{63} & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\gamma_6 = s_{61}.\sigma_1 = 0 \rightarrow s_{61}=0 \rightarrow s_{16}=0$$

Benzer şekilde, sadece σ_2 veya σ_3 varken $s_{62}=0 \rightarrow s_{26}=0$, $s_{63}=0 \rightarrow s_{36}=0$ bulunur.

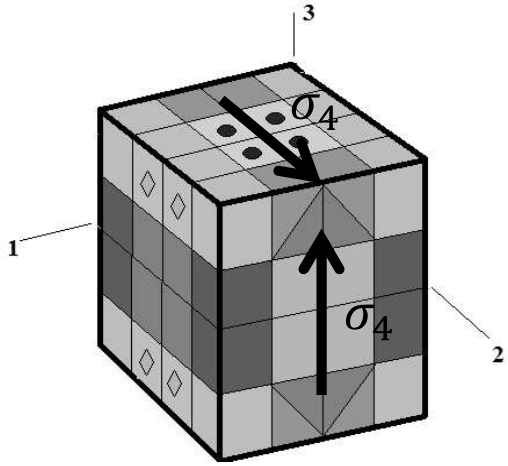
5.6 Ortotropik Malzeme ve Elastik Malzeme Sabitleri -devam

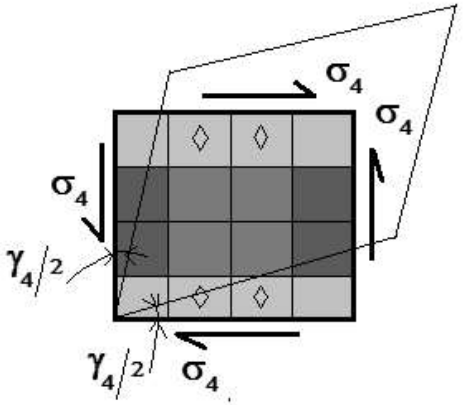
(orthotropic material and elastic material constants -continue)



Ayrıca malzemeye sadece $\sigma_{23} = \tau_4 = \sigma_4$ uygulanırsa $\gamma_4 = S_{44}.\sigma_4$

Simetriden dolayı diğer düzlemlerde bir kayma açısı oluşmaz. $\gamma_5 = \gamma_6 = 0$





$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_4 \\ \gamma_5(=0) \\ \gamma_6(=0) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{54} & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sigma_4 \\ 0 \\ 0 \end{bmatrix}$$

$\gamma_5 = S_{54}.\sigma_4 = 0 \rightarrow S_{54} = 0 \rightarrow S_{45} = 0$ bulunur.

Bu durumda, ortotropik malzeme için gevşeklik ve rijitlik matrisleri:

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \qquad [C]=\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

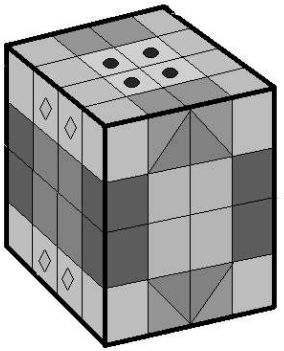
Sonuç olarak ;
ortotropik bir malzeme için bağımsız malzeme sabitleri 9 tanedir.

$$E_1, E_2, E_3, \quad G_{23}, G_{13}, G_{12}, \quad \upsilon_{23}, \upsilon_{13}, \upsilon_{12}$$

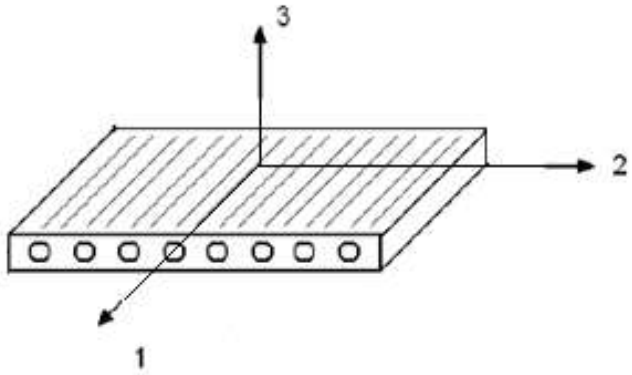
$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\upsilon_{21}}{E_2} & -\frac{\upsilon_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\upsilon_{12}}{E_1} & \frac{1}{E_2} & -\frac{\upsilon_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\upsilon_{13}}{E_1} & -\frac{\upsilon_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

Fiber takviyeli kompozit bir tabaka , makro seviyede 3 düzleme göre de simetriktir ve makro seviyede ortotropik malzeme olarak düşünülebilirler.

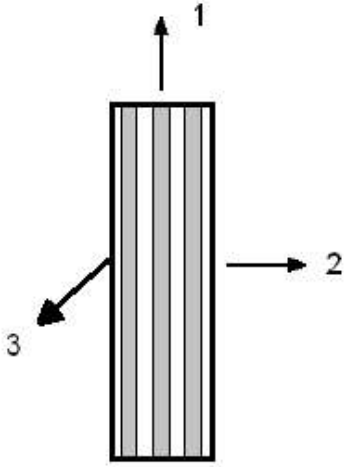
. (The fiber reinforced laminated composite plates have 3 symmetry plane at macro level)



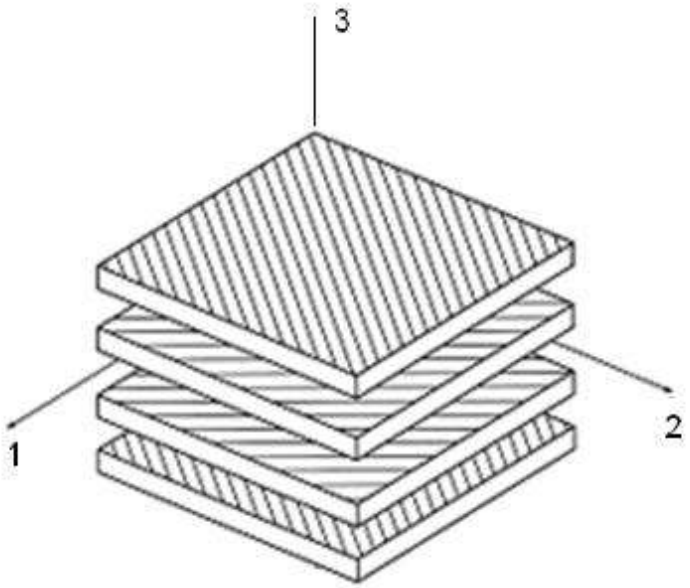
orthotropic materials



Ancak malzeme özellikleri tespit edilirken, tabakadan çıkarılacak test numunesinin de makro seviyede 3 düzleme göre simetrik olmasına özen gösterilmelidir. (But, the test speciements obtained from the laminated composites should be also symettry to these three planes)



Tabakalı kompozitler bu simetrikliği makro seviyede sağlıyorsa, aynı şekilde ortrotropik malzeme olarak kabul edilirler. (If the composites provide the symmetry to the 3 planes, they are accepted to orthotropric materials)

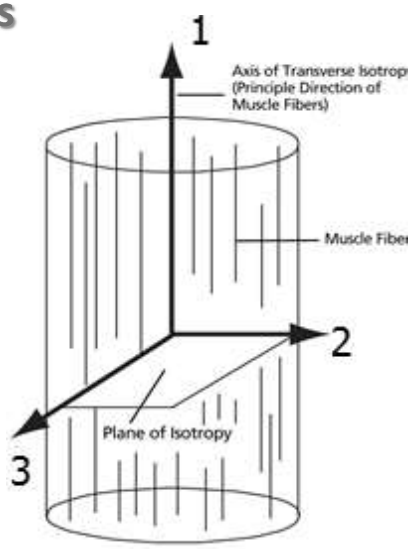


Soru: yanda görülen, 2 mm kalınlıkta 4 tabakadan oluşturulmuş kompozit yapıdan çıkarılacak çekme numunesinin kalınlığı minimum ne olmalıdır?

Question: What is the minimum thickness of tensile speciement obtained from the laminated composite, shown the figure, having the 4 layer. Each layer has the thickness 2mm

5.7 Enine (Transversely) Izotropik Malzeme ve Elastik Malzeme Sabitleri

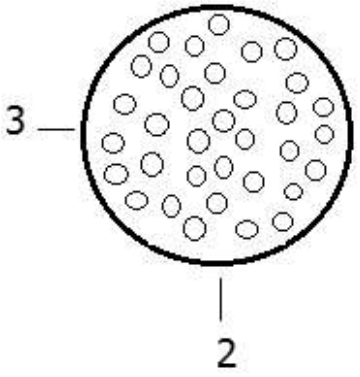
Transversely Isotropic Material and its Elastic Material Constants



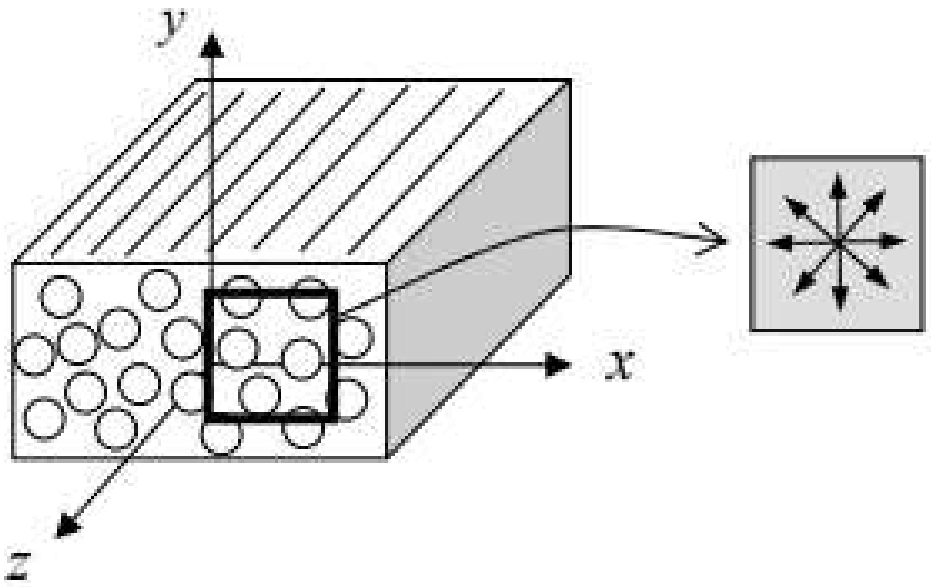
Eğer ortotropik malzemenin her noktasında, birbirlerine paralel olacak şekilde izotropi düzlemleri varsa, bu malzemelere enine izotropik (transversely isotropic) malzemeler denir.

(If there are also symmetry of isotropy planes, each of which is parallel to each others, at all point of orthotropic materials, this type of materials are called transversly isotropic.)

Malzeme özellikleri o düzlemde her doğrultuda aynı olmasına rağmen düzleme dik doğrultuda farklılık gösterir. (The material properties are the same along to the all directions at this isotropy planes)



Fiber eksenine (1'e) göre dönel simetriye sahip kompozit elemanlar enine izotropik malzemelerdir. Zira 2-3 düzleminde yaklaşık her doğrultuda özellikler aynıdır. 1 doğrultusunda farklıdır. (The composite having the axisymmetry about to the fiber axis are transverly isotropic materials. The properties at the plane 2-3 are appoximetly same along to the all directions but they are different along to the fiber axis -1)



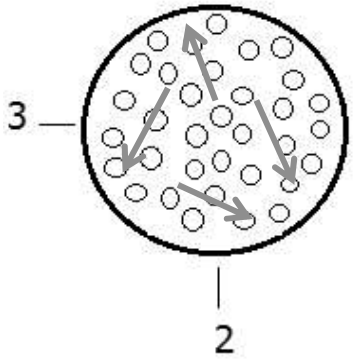
Fiberlerin karışık dağılımda yerleştirildiği prizmatik bir kompozit hacimde, 1-2 düzlemi içindeki ve ona paralel tüm düzlemler içindeki tüm doğrultularda özellikler aynı kabul edilebilir ve bu durumda bu tip bir yapı da enine (transversely) izotropiktir.

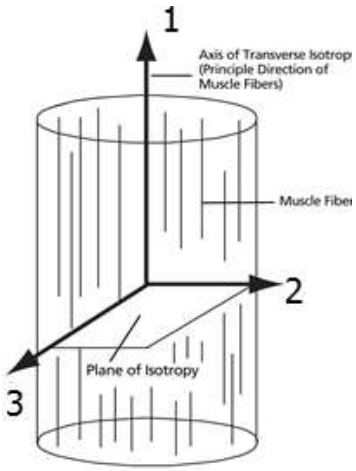
5.7 Enine (Transversely) Izotropik Malzeme Örneği

An example for Transversely Isotropic Materials



Transverse Isotropy is observed in sedimentary rocks at long wavelengths. Each layer has approximately the same properties in-plane but different properties through-the-thickness. The plane of each layer is the plane of isotropy and the vertical axis is the axis of symmetry.





Dönel simetriye sahip, fiber takviyeli kompozit malzemelerde dik kesitte (2-3 düzleminde), tüm yönlerde malzeme özellikleri aynı kabul edilebilir:

(İzotropik malzeme özelliği)

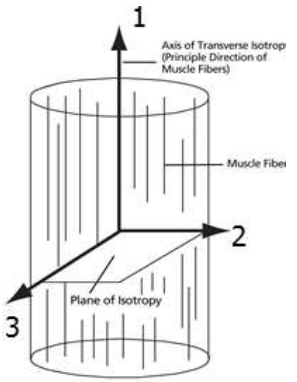
$$E_3 = E_2 \quad , \quad G_{13} = G_{12} \quad , \quad \upsilon_{13} = \upsilon_{12} \quad , \quad G_{23} = \frac{E_2}{2(1+\upsilon_{23})} \ldots (I)$$

Böylece bağımsız malzeme sabiti 9 dan 5 e indirgenmiş olur.
(E_2 ve ν_{23} belli iken (I) formülünden bulunabilmektedir. G_{23})

1. E_1
2. $E_2 (= E_3)$
3. $G_{13} (= G_{12})$
4. υ_{23}
5. $\upsilon_{13} (= \upsilon_{12})$

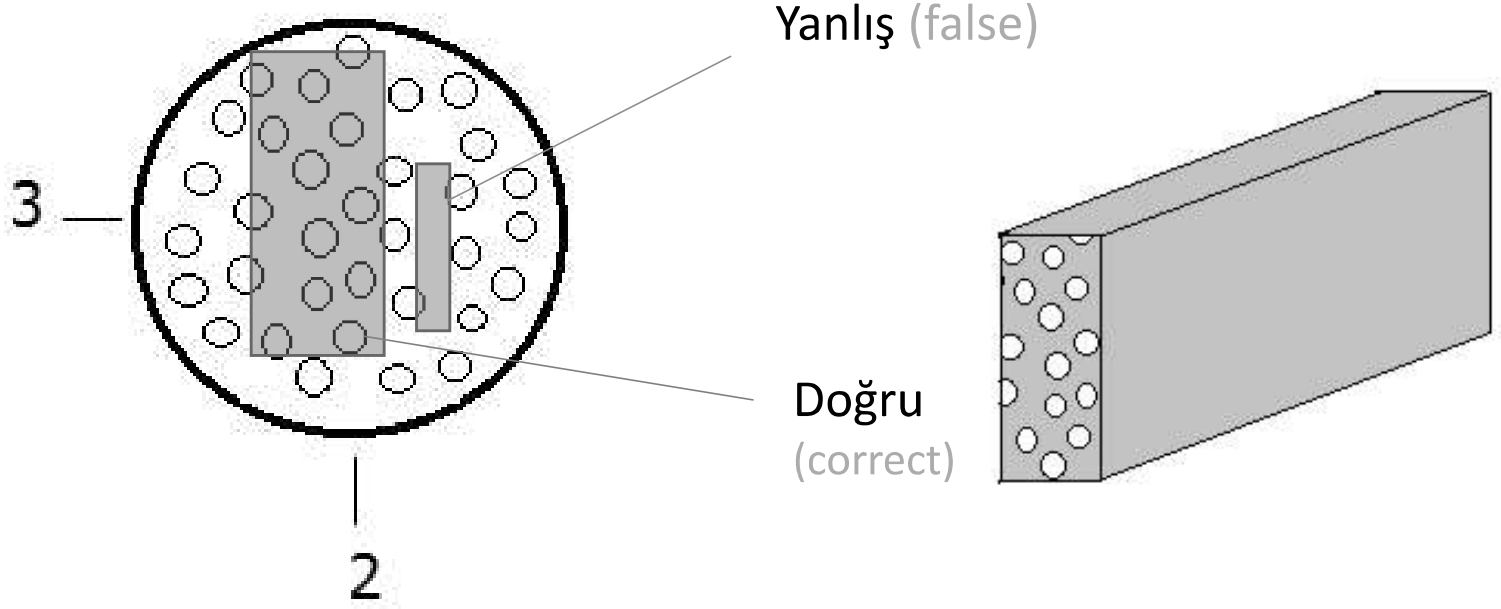
$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{bmatrix}=\begin{bmatrix}S_{11} & S_{12} & S_{13} & 0 & 0 & 0\\ S_{12} & S_{11} & S_{13} & 0 & 0 & 0\\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & S_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & S_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & 2(S_{11}-S_{12})\end{bmatrix}\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{bmatrix}$$

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0\\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & (C_{11}-C_{12})/2\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{bmatrix}$$



Çekme numunesi enine boyutları yeterince büyük olmalı tüm yapıyı temsil edebilmelidir. :

Tensile Test speciement



5.8 İzotropik Malzeme ve Elastik Malzeme Sabitleri

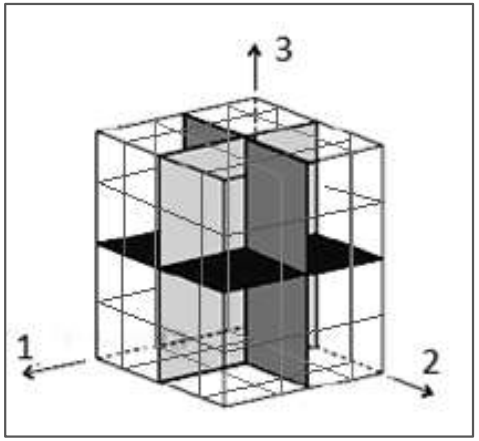
Isotropic material and its elastic material constants

Malzeme içinde seçilen tüm düzlemlerde, tüm doğrultularda özellikler aynı ise bu tip malzemelere izotropik malzemeler denir. (If the properties at any plane and any directions are same, this materials are called isotropic)
Yani malzeme içinde tüm doğrultularda sonsuz izotropik simetri düzlemi vardır.
Bu durumda elastik malzeme sabitleri 5 den 2'ye (E,ν) indirgenmiş olur: (The material constants are reduced from 5 to 2: E,ν)

$$E_1 = E_2 = E_3 = E \qquad G_{23} = G_{13} = G_{12} = G \qquad \upsilon_{23} = \upsilon_{13} = \upsilon_{12} = \upsilon \qquad G = \frac{E}{2(1+\upsilon)}$$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(S_{11}-S_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(S_{11}-S_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(S_{11}-S_{12}) \end{bmatrix} \quad [C] = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} \end{bmatrix}$$

1-2, 2-3 ve 1-3 düzlemlerinde ve onlara paralel tüm düzlemlerde sonsuz tane izotropi düzlemi vardır.



$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{bmatrix}=\begin{bmatrix}\frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0\\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0\\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{2(1+\upsilon)}{E} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{2(1+\upsilon)}{E} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\upsilon)}{E}\end{bmatrix}\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{bmatrix}$$

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}\frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0\\ \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0\\ \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & \frac{\upsilon E}{(1-2\upsilon)(1+\upsilon)} & \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{E}{2(1+\upsilon)}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{bmatrix}$$